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International Journal of Solids and Structures 41 (2004) 295–304

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

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A statistical micromechanics-based multi-scale framework for effective thermomechanical behaviours of particle reinforced composites

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Received 12 September 2003; received in revised form 18 September 2003

Abstract

Particle-reinforced composite materials have been widely used as they can exhibit nearly isotropic material properties and are often easy to process. Some silicon carbide particle reinforced aluminium composites with high volume concentration of reinforcement exhibit excellent thermophysical properties and can be used in advanced electronic packaging. In this paper, a statistical micromechanics-based multi-scale material modelling framework is introduced to describe the macroscopic effective thermomechanical properties of the particle-reinforced composite. The formulation differs from most of the existing methods in that the interaction effects among the reinforcing particles are directly accounted for by considering pair-wise interaction and statistical information on particle distribution is included. The strain and stress concentration factor tensors that relate the local average strain and stress fields, respectively, to the corresponding global average fields are derived according to the theory of average fields. The effective coefficient of thermal expansion for the particle-reinforced composite material is derived. Comparisons of the prediction from the proposed framework to the results from other existing methods are presented. The results are expressed in analytical closed-form in terms of the thermal and mechanical properties of the two constituent phases and the volume fraction of particles. No parameter estimation or data fitting is required.

Keywords: Composite materials; Thermoconductivity; Micromechanics; Multi-scale material modelling; Constitutive equation

1. Introduction

The study of the effective material behaviours of composite materials has been an important subject due to the wide range of applications of advanced composite materials in various engineering disciplines including civil constructions, aerospace structures, and the automobile industry. A comprehensive literature review on this topic can be found in Tseng (1995). Applications of particle reinforced composites to advanced technologies such as electronic packaging and smart materials are emerging. Recently, Kim et al.

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0020-7683/$ - see front matter
doi:10.1016/j.ijsolstr.2003.09.035

(2001) investigated the effective thermal expansion coefficient for silicon carbide particle reinforced aluminium composites. A study on the thermomechanical behaviour of porous shape memory alloys can be found in Qidwai et al. (2001). In this paper, the focus is on the effective thermomechanical behaviours of general particle reinforced composite materials.

To simplify the derivation, all reinforcing particles are assumed spherical in shape, equal in size, and embedded firmly in the matrix material. Both the matrix material and the reinforcing particles are within their elastic limits under the loading condition considered in this work. The particles are distributed randomly among the matrix material. Nevertheless, the proposed framework is capable of modelling more general problems including the variation of particle size, shape, and the distribution function.

The theory of average fields is introduced as a tool to link the local average stress and strain fields to the corresponding global averages. Equations relating the local average to the global ones are written in terms of the stress and strain concentration factor tensors. A statistical micromechanics-based framework is then applied to derive the stress and stain concentration factor tensors. Since the particles are assumed to be distributed randomly among the matrix material, it is impractical to consider only a given realisation of the distribution of particles. Furthermore, the reinforcing particles could be close to each other in the case of moderately high to high concentration of particles in the composite. The interaction effects among the particles are therefore an important factor when considering the macroscopic properties of the composite. However, considering the effects of interaction among all the particles is intractable due to the large number of particles typically being added to the composite. Under all the condition, the effect of interaction between two particles is calculated analytically by applying the law of mechanics at the microscopic level. The pairwise interaction solution is then averaged among the statistical space to account for the inhomogeneity of the composite. This leads to an approximate yet closed-form analytical results.

The formulation starts from applying the concept of eigenstrain via the Eshelby's equivalence principle to solve the problem of two spherical particles embedded firmly in an infinite elastic solid. Approximate yet closed-form solutions are derived for this particle-interaction problem. Based on the solutions, an ensemble average is performed to account for the statistical distribution of the particles among the matrix material. Furthermore, a volume average is applied to obtain the global effective properties. The stress and strain concentration factor tensors for the proposed framework are identified. The average stress and strain fields for particle-reinforced composite materials under different loading conditions can be obtained for both of the cases that with and without considering the inter-particle-interaction effect.

Based on the concept of the theory of average fields, the macroscopic thermal expansion coefficient for the particle-reinforced composite is derived through the stress and strain concentration factor tensors. The effect of thermal–mechanical interaction is included in the results via the theory of average fields. The derived thermal expansion coefficient differs from the results from other existing approaches in that the interaction effects among the reinforcing particles are directly accounted for and explicit closed-form analytical expressions are obtained. In addition, the proposed framework has the capability of accounting for the statistical information on the distribution of the reinforcing particles in the composite.

Based on the explicit expression obtained in this study, numerical results from the proposed framework are compared with the predictions from other existing formulations available in the literature. Finally, directions for further research based on the proposed framework are pointed out. Future research work including incorporating the proposed model into the non-linear finite element analysis for boundary value problems in practical engineering applications is outlined.

This paper is organised in the following fashion. Section 1 contains an introduction of the framework proposed in this study. Then, the problem of two equal-sized spherical particles imbedded firmly in an infinitely extended isotropic elastic solid is solved in Section 2 using the concept of distributed eigenstrain and the Eshelby's equivalence principle. In Section 3, the average field theory is introduced and the stress and strain concentration factor tensors for the particle reinforced composite material are derived based on the solution in Section 2. As an illustrative example for the application of the stress and strain concentration tensors, the effective elastic moduli for the particle reinforced composite are expressed in explicit closed-form. Section 5 gives the explicit expression of the effective coefficient of thermal expansion for the composite considered and compares the prediction with the results from existing approaches. The paper is concluded in Section 6.

2. Interaction of two spherical inclusions

Let us consider the problem of two spherical inclusions embedded firmly in an infinite elastic solid subjected to a far field loading. For simplicity, it is assumed that the two spherical inclusions are of the same size and their radius is denoted as r. As shown in Fig. 1, the locations for the centres of spherical particle 1 and 2 are denoted as $\mathbf{x}_1$ and $\mathbf{x}_2$, respectively. Apparently, there are two material phases in this problem. Phase 0 and 1 denotes the matrix and particle phase, respectively. Furthermore, Ω_1 and Ω_2 represent the domain inside of particle 1 and 2. The vector $\mathbf{r}$ denotes the relative position between the two centres.

The framework that is proposed in this paper is valid for the general composite system with any arbitrary material property for the constituent phase. However, for the simplicity of presentation and mathematical operation, we assume that the material properties for both the matrix phase and the particle phase are isotropic and the loading at any local material point remains within the elastic limit. It is further assumed that the particles do not intersect each other and the material properties of both phases remain unchanged for the loading considered.

Following the procedure detailed in Tseng (1995), when applying the Eshelby's equivalence principle to the inclusion problem without considering the effects of inter-particle interaction, the equation for determining the unknown eigenstrain, which has been proved to be constant throughout the entire spherical region, can be written as

$$-\mathbf{A} : \boldsymbol{\varepsilon}^{*0} = \boldsymbol{\varepsilon}^{0} + \mathbf{S} : \boldsymbol{\varepsilon}^{*0} \tag{1}$$

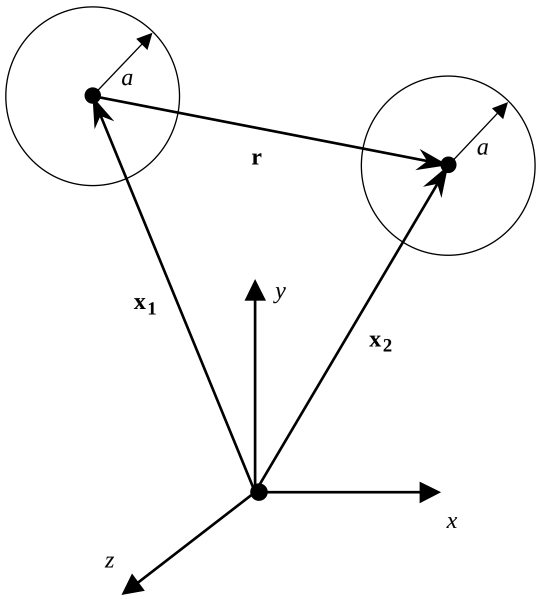

Fig. 1. Two interacting spherical particles.

where

$$\mathbf{A} = (\mathbf{C}_1 - \mathbf{C}_0)^{-1} \cdot \mathbf{C}_0 \tag{2}$$

in which $\mathbf{C}_0$ and $\mathbf{C}_1$ are the stiffness tensor for the matrix and inclusion phase, respectively. In Eq. (1), $\mathbf{S}$ is the Eshelby's tensor for a spherical inclusion and is defined as

$$\mathbf{S} = \int_{\Omega} \mathbf{G}(\mathbf{x} - \mathbf{x}')\mathrm{d}\mathbf{x}', \quad \mathbf{x} \in \Omega \tag{3}$$

where the tensor $\mathbf{G}(\mathbf{x} - \mathbf{x}')$ is the Green's function for elasticity and is defined by the following equation

$$\boldsymbol{\varepsilon}(\mathbf{x}) = \int \mathbf{G}(\mathbf{x} - \mathbf{x}') : \boldsymbol{\varepsilon}^*(\mathbf{x}')\mathrm{d}\mathbf{x}' \tag{4}$$

in which $\boldsymbol{\varepsilon}(\mathbf{x})$ denotes the strain tensor at the location $\mathbf{x}$, $\boldsymbol{\varepsilon}^*(\mathbf{x})$ is the tensor of eigenstrain, and $\boldsymbol{\varepsilon}^{*0}(\mathbf{x})$ represents the eigenstrain tensor for the non-interacting particles. The explicit form for the tensor components of $\mathbf{S}$ can be found in Mura (1987) for the spherical inclusion considered in the present study. The Eshelby's tensor for other shapes of inclusion can be found in Mura (1987). Taking into account the effects of inter-particle interaction, the integral equation governing the distributed eigenstrain can be expressed as

$$-\mathbf{A} : \boldsymbol{\varepsilon}^*(\mathbf{x}) = \boldsymbol{\varepsilon}^0 + \int_{\Omega_i} \mathbf{G}(\mathbf{x} - \mathbf{x}') : \boldsymbol{\varepsilon}(\mathbf{x}')\mathrm{d}\mathbf{x}' + \int_{\Omega_j} \mathbf{G}(\mathbf{x} - \mathbf{x}') : \boldsymbol{\varepsilon}(\mathbf{x}')\mathrm{d}\mathbf{x}' \tag{5}$$

With the effect of inter-particle interaction, the distributed eigenstrain in both of the spherical inclusions is no longer uniform. To capture and average amount of perturbation on the eigenstrain from the non-interacting case, i.e., $\boldsymbol{\varepsilon}^{*0}(\mathbf{x})$ and following the procedure in Tseng (1995), the following equation can be obtained after dropping the higher order terms for the parameter $\rho = r/a$ where $r = |\mathbf{r}|$:

$$-\mathbf{A} : \mathbf{d}^* = \mathbf{G}^2(\mathbf{x}_1 - \mathbf{x}_2) : \boldsymbol{\varepsilon}^{*0} + \mathbf{S} : \mathbf{d}^* + \mathbf{G}^1(\mathbf{x}_1 - \mathbf{x}_2) : \mathbf{d}^* \tag{6}$$

The tensors in Eq. (6) are defined as

$$\mathbf{d}^* = \frac{1}{\Omega} \int_{\Omega} [\boldsymbol{\varepsilon}^*(\mathbf{x}) - \boldsymbol{\varepsilon}^{*0}]\mathrm{d}\mathbf{x} \tag{7}$$

$$\mathbf{G}^1(\mathbf{x}_1 - \mathbf{x}_2) = \int_{\Omega_1} \mathbf{G}(\mathbf{x} - \mathbf{x}_2)\mathrm{d}\mathbf{x} = \int_{\Omega_2} \mathbf{G}(\mathbf{x}_1 - \mathbf{x})\mathrm{d}\mathbf{x} \tag{8}$$

and

$$\mathbf{G}^2(\mathbf{x}_1 - \mathbf{x}_2) = \frac{1}{\Omega} \int_{\Omega_1} \int_{\Omega_2} \mathbf{G}(\mathbf{x} - \mathbf{x}')\mathrm{d}\mathbf{x}'\,\mathrm{d}\mathbf{x} \tag{9}$$

See Tseng (1995) for the explicit closed-form expression for the components of the above-mentioned tensors.

Let us now consider the case that many equal-sized spherical particles distributed randomly among an elastic solid. Based on the solution for Eq. (6), which represents the effect of pair-wise interaction, and assuming that the distribution of the particles is uniform and no particle overlaps with each other, ensemble-volume averaged eigenstrain perturbation in a particle can be written as

$$\langle \bar{\boldsymbol{\varepsilon}}^* \rangle = \boldsymbol{\Gamma} : \boldsymbol{\varepsilon}^{*0} \tag{10}$$

where the components for the isotropic *interaction tensor* $\boldsymbol{\Gamma}$ are defined as

$$\Gamma_{ijkl} = \gamma_1 \delta_{ij}\delta_{kl} + \gamma_2(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \tag{11}$$

in which δ_{ij} is the Kronecker delta,

$$\gamma_1 = \frac{5\phi}{4\beta^2}\left\{-2(1-\nu_0) - 5\nu_0^2 - \frac{4\alpha}{3\alpha+2\beta}(1+\nu_0)(1-2\nu_0)\right\} \tag{12}$$

and

$$\gamma_2 = \frac{1}{2} + \frac{5\phi}{8\beta^2}\left\{11(1-\nu_0) + 5\nu_0^2 - \frac{3\alpha}{3\alpha+2\beta}(1+\nu_0)(1-2\nu_0)\right\} \tag{13}$$

where

$$\alpha = 2(5\nu_0 - 1) + 10(1-\nu_0)\left(\frac{\kappa_0}{\kappa_1-\kappa_0}\right)\left(\frac{\mu_0}{\mu_1-\mu_0}\right) \tag{14}$$

and

$$\beta = 2(4-5\nu_0) + 15(1-\nu_0)\frac{\mu_0}{\mu_1-\mu_0} \tag{15}$$

In Eqs. (12) and (13), ϕ denotes the volume fraction of the particles in the composite material under consideration. In addition, ν, κ, and μ represent the Poisson ratio, bulk modulus, and shear modulus, respectively, for the corresponding material phase which is denoted via the corresponding subscript. Subscript 0 is for the matrix phase and subscript 1 denotes the particle phase. For simplicity, both the matrix phase and the particle phase are assumed to be isotropic and the loading applied is within their elastic limits.

It is evident from Eq. (10) that if the interaction tensor $\mathbf{\Gamma}$ is set to be equal to the identity tensor $\mathbf{I}$, which means that, in indicial notation,

$$\Gamma_{ijkl} = I_{ijkl} = \frac{1}{2}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \tag{16}$$

then, the formulation recovers the situation that the effect of inter-particle interaction is neglected.

3. Concentration factor tensors

Due to the high degree of complexity of the arbitrary geometry and concentration of the reinforcing material, the determination of the exact internal local stress or strain field in a composite system is in general formidable. In many applications, it is sufficient provided that the average of the field concerned is available. A method based on the so called stress and strain concentration factors was introduced by Hill (1963) and later extended by Dvorak (1991) to address the effective properties of composite materials.

In physical sense, the concentration factor defines the relationship between the local field and the average of the global field. In the case that the stress field is in concern, the stress at any local point for a specific material phase is related to the average stress for the global composite system via the stress concentration factor. If only the average of the local stress field is required, upon averaging over the local material phase, we can obtain the following relationship

$$\overline{\boldsymbol{\sigma}}_\alpha = \mathbf{B}_\alpha : \overline{\boldsymbol{\sigma}} \tag{17}$$

where the fourth rank tensor $\mathbf{B}_\alpha$ is the volume averaged stress concentration factor tensor for phase α, an over-bar represents the volume average of the corresponding quantity, and the subscript α denotes the material phase. Similar definition is made for the strain field.

$$\overline{\boldsymbol{\varepsilon}}_\alpha = \mathbf{A}_\alpha : \overline{\boldsymbol{\varepsilon}} \tag{18}$$

in which $\mathbf{A}_\alpha$ is the volume averaged strain concentration factor tensor for phase α.

Since the two material phases are assumed not to overlap each other, the averaging process at the global scale can be separated into two parts—one for each phase. Therefore, the following two equations can be obtained

$$\overline{\boldsymbol{\sigma}} = \phi_0\overline{\boldsymbol{\sigma}}_0 + \phi_1\overline{\boldsymbol{\sigma}}_1 \tag{19}$$

$$\overline{\boldsymbol{\varepsilon}} = \phi_0\overline{\boldsymbol{\varepsilon}}_0 + \phi_1\overline{\boldsymbol{\varepsilon}}_1 \tag{20}$$

with Eqs. (17) and (18), the relationship between the concentration factors for the two phases can be written as

$$\phi_0\mathbf{B}_0 + \phi_1\mathbf{B}_1 = \mathbf{I} \tag{21}$$

$$\phi_0\mathbf{A}_0 + \phi_1\mathbf{A}_1 = \mathbf{I} \tag{22}$$

Moreover, the elastic stiffness and compliance tensors, $\mathbf{C}_\alpha$ and $\mathbf{D}_\alpha$, respectively, for material phase α relate the local average stress and strain fields according to the following two equations

$$\overline{\boldsymbol{\sigma}}_\alpha = \mathbf{C}_\alpha : \overline{\boldsymbol{\varepsilon}}_\alpha \tag{23}$$

$$\overline{\boldsymbol{\varepsilon}}_\alpha = \mathbf{D}_\alpha : \overline{\boldsymbol{\sigma}}_\alpha \tag{24}$$

Similarly, the macroscopic elastic properties can be expressed by the following equations through the global elastic moduli

$$\overline{\boldsymbol{\sigma}} = \mathbf{C}_* : \overline{\boldsymbol{\varepsilon}} \tag{25}$$

$$\overline{\boldsymbol{\varepsilon}} = \mathbf{D}_* : \overline{\boldsymbol{\sigma}} \tag{26}$$

Consequently, from Eqs. (17)–(26), the global effective elastic moduli are expressed in terms of the volume fractions, elastic moduli of the constituent phases, and the concentration factor tensors as shown in the following two equations

$$\mathbf{C}_* = \phi_0\mathbf{C}_0 \cdot \mathbf{A}_0 + \phi_1\mathbf{C}_1 \cdot \mathbf{A}_1 \tag{27}$$

$$\mathbf{D}_* = \phi_0\mathbf{D}_0 \cdot \mathbf{B}_0 + \phi_1\mathbf{D}_1 \cdot \mathbf{B}_1 \tag{28}$$

More concise and convenient forms which depends on quantities related to a single material phase can be derived with the help of Eqs. (21) and (22):

$$\mathbf{C}_* = \mathbf{C}_\alpha + \phi_\beta(\mathbf{C}_\beta - \mathbf{C}_\alpha) \cdot \mathbf{A}_\beta \tag{29}$$

$$\mathbf{D}_* = \mathbf{D}_\alpha + \phi_\beta(\mathbf{D}_\beta - \mathbf{D}_\alpha) \cdot \mathbf{B}_\beta \tag{30}$$

From Eqs. (29) and (30), the global effective elastic moduli for a two-phase composite system can be obtained provided that any stress or strain concentration factor tensor is available.

As illustrated in Tseng (1995), the equation relating the average strain $\overline{\boldsymbol{\varepsilon}}$, the uniform remote strain $\boldsymbol{\varepsilon}^0$, and the average eigenstrain $\overline{\boldsymbol{\varepsilon}}^*$ can be expressed as:

$$\overline{\boldsymbol{\varepsilon}} = \boldsymbol{\varepsilon}^0 + \phi\mathbf{S} : \overline{\boldsymbol{\varepsilon}}^* \tag{31}$$

With Eqs. (31), (10) and (1), we get

$$\overline{\boldsymbol{\varepsilon}}^* = \mathbf{B} : \overline{\boldsymbol{\varepsilon}} \tag{32}$$

where

$$\mathbf{B} = \boldsymbol{\Gamma} \cdot [-\mathbf{A} - \mathbf{S} + \phi\mathbf{S} \cdot \boldsymbol{\Gamma}]^{-1} \tag{33}$$

Averaging the fundamental equation for the Eshelby's equivalence principle:

$$\mathbf{C}_1 : \boldsymbol{\varepsilon}(\mathbf{x}) = \mathbf{C}_0 : [\boldsymbol{\varepsilon}(\mathbf{x}) - \boldsymbol{\varepsilon}^*(\mathbf{x})] \tag{34}$$

the relationship between the local strain average and the eigenstrain average can be written as

$$\mathbf{C}_1 : \overline{\boldsymbol{\varepsilon}}_1 = \mathbf{C}_0 : [\overline{\boldsymbol{\varepsilon}}_1 - \overline{\boldsymbol{\varepsilon}}^*] \tag{35}$$

Further utilising Eq. (2), we arrive at

$$\overline{\boldsymbol{\varepsilon}}_1 = \mathbf{A} : \overline{\boldsymbol{\varepsilon}}^* \tag{36}$$

then, with Eq. (31),

$$\overline{\boldsymbol{\varepsilon}}_1 = -(\mathbf{A} \cdot \mathbf{B}) : \overline{\boldsymbol{\varepsilon}} \tag{37}$$

Hence, upon comparing with Eq. (37) with Eq. (18), the strain concentration factor tensor considering the effect of inter-particle interaction can be written as

$$\mathbf{A}_1 = -\mathbf{A} \cdot \mathbf{B} \tag{38}$$

and the corresponding stress concentration factor tensor can be derived by using Eqs. (27) and (28). The explicit expression for the stress concentration factor tensor takes the following form

$$\mathbf{B}_1 = -\mathbf{C}_1 \cdot \mathbf{A} \cdot \mathbf{B} \cdot [\mathbf{I} - \phi\mathbf{B}]^{-1} \cdot \mathbf{C}_0^{-1} \tag{39}$$

The tensor components for $\mathbf{B}_1$ can be obtained by carrying out the lengthy tensor operation in Eq. (39). In this study, to ensure the correctness of the formulation, two symbolic mathematical manipulation software, Maple and Mathematica, have been used to facilitate the derivation of the complex analytical expression and operation. The fourth rank tensor $\mathbf{B}_1$ is found to be isotropic and its components are

$$(B_1)_{ijkl} = b_1\delta_{ij}\delta_{kl} + b_2(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \tag{40}$$

where

$$3b_1 + 2b_2 = \frac{30\kappa_1(1 - v_0)(3\gamma_1 + 2\gamma_2)}{(\kappa_0 - \kappa_1)[(3\alpha + 2\beta) + 20(1 - 2v_0)(3\gamma_1 + 2\gamma_2)]} \tag{41}$$

and

$$b_2 = \frac{15\mu_1(1 - v_0)\gamma_2}{(\mu_1 - \mu_0)[\beta + 2(7 - 5v_0)\phi\gamma_2]} \tag{42}$$

4. Effective elastic properties

As an example, the stress and strain concentration factor tensors derived in the previous section are employed to construct the effective elastic properties for particle reinforced composites. Through Eqs. (29) and (39), the effective elastic stiffness tensor incorporating the effect of inter-particle interaction reads

$$\mathbf{C}_* = \mathbf{C}_0 \cdot \{\mathbf{I} - \phi\boldsymbol{\Gamma} \cdot (-\mathbf{A} - \mathbf{S} + \phi\mathbf{S} \cdot \boldsymbol{\Gamma})^{-1}\} \tag{43}$$

It is noted that Eq. (43) recovers the results from the Mori–Tanaka method if the effect of inter-particle interaction is neglected, i.e., letting $\boldsymbol{\Gamma} \to \mathbf{I}$ or equivalently setting $\gamma_1 \to 0$ and $\gamma_2 \to 1/2$.

Since the particles are assumed to be distributed uniformly among the matrix material, the composite is isotropic. Therefore, the effective elastic property can be represented by the effective bulk modulus κ_* and the effective shear modulus μ_* can be explicitly written as

$$\kappa_* = \kappa_0 \left\{ 1 + \frac{30(1-\nu_0)\phi(3\gamma_1 + 2\gamma_2)}{3\alpha + 2\beta - 10(1+\nu_0)\phi(3\gamma_1 + 2\gamma_2)} \right\} \tag{44}$$

and

$$\mu_* = \mu_0 \left\{ 1 + \frac{30(1-\nu_0)\phi\gamma_2}{\beta - 4(4-5\nu_0)\phi\gamma_2} \right\} \tag{45}$$

5. Effective thermal conductivity

The effective thermal conductivity for the particle-reinforced composite can be obtained via the pure mechanical concentration factor tensors, i.e., the stress and strain concentration factor tensors in the present study. For detailed discussion, please see Dvorak and Chen (1992).

Assuming that the constitutive equation of thermoelasticity for phase α is given as

$$(\sigma_\alpha)_{ijkl} = \lambda_\alpha \delta_{ij} \varepsilon_{kk} + 2\mu \varepsilon_{ij} - \beta_\alpha \delta_{ij} \theta \tag{46}$$

where θ denotes the temperature and the volumetric thermal expansion coefficient is defined as

$$\beta_\alpha = 3\alpha_\alpha \kappa_\alpha \tag{47}$$

in which α_α is the linear thermal expansion coefficient for phase α. Similar to the notation used for the effective elastic properties, a subscript of * represents the quantity corresponding to the composite.

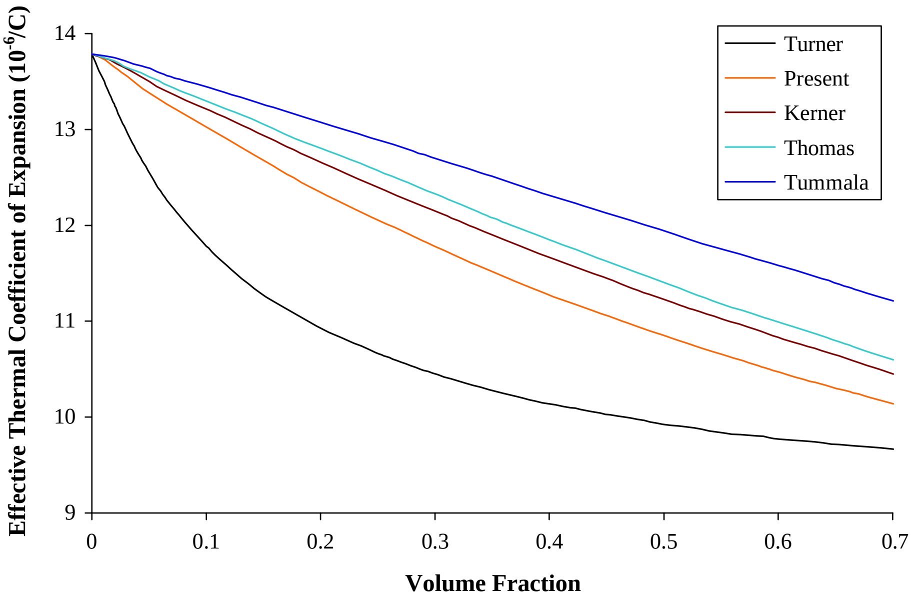

Fig. 2. Comparison with other existing results.

Table 1
Material properties for the numerical example

	α (10^{-6}/°C)	E (10^6 psi)	v
Matrix	13.80	6.8	0.13
Particle	9.40	28.0	0.30

Furthermore, let us assume that the second rank tensors $\mathbf{b}_0$ and $\mathbf{b}_1$ denote the thermal stress for the matrix and the particle phase, respectively. The tensor components are

$$(b_\alpha)_{ij} = -\beta_\alpha \delta_{ij} = -3\alpha_\alpha \kappa_\alpha \delta_{ij} \tag{48}$$

Using the stress and strain concentration factor tensors, the effective coefficient of thermal expansion can be written as

$$\alpha_* = \alpha_1 \frac{\kappa_1}{\kappa_*} + \frac{1}{\kappa_*}(\alpha_1 \kappa_1 - \alpha_0 \kappa_0)\frac{\kappa_* - \kappa_1}{\kappa_1 - \kappa_0} \tag{49}$$

where the effective bulk modulus κ_* for the particle reinforced composite is given in Eq. (44).

The results in Eq. (49) incorporate the effect of inter-particle interaction. Fig. 2 shows the comparison of the prediction based on Eq. (49) with the other models discussed in Tummala and Friedberg (1970). In the numerical calculation, the material properties in Table 1 are utilised.

From Fig. 2, the prediction from the present study show similar trend with the results from the other models and lies within the curves for the methods proposed by Kerner and Turner. The thermal expansion coefficient decreases as the volume fraction of the particle increases. This is consistent with the physical sense since the particle phase has a smaller thermal expansion coefficient.

6. Conclusions

In summary, this paper presents a framework for the theoretical prediction on effective macroscopic thermal–mechanical properties for particle-reinforced composite materials. Closed-form and analytical explicit expressions for the effective coefficient of thermal conductivity are derived. Interaction effects among the reinforcing particles are accounted for by considering the pair-wise interaction between two particles. The ensemble-volume averaging process is applied to account for the statistical distribution of particles. The theory of average fields is employed to relate the local average fields to the global averages via the stress and strain concentration factor tensors. Furthermore, the stress and strain concentration factor tensors are derived for the proposed framework. Finally, the macroscopic effective coefficient of thermal conductivity is obtained. Comparisons of the predictions from the proposed framework with other existing methods are presented. The effect of inter-particle interaction on the thermal coefficient of expansion is accounted for directly through taking the statistical average on the solution of the pair-wise inter-particle interaction problem. This is the major difference between the present study and other existing models. In addition, the present framework is capable of capturing the statistical distribution on the locations of the reinforcing particles. The formulation recovers the non-interacting solution if the effects of inter-particle interaction are neglected.

Further research work is undertaken by the author to incorporate the effective thermal coefficient of expansion to the linear and non-linear finite element analysis. The approach is similar to the work in Tseng (1995) where the effective elastic and elasto–visco-plastic material properties are implemented into non-linear and time-dependent finite element analysis to solve boundary value problems. The results of the present study will enable the numerical analysis of the thermomechanical behaviours of particle reinforced composite materials via finite element analysis. Boundary value problems can be solved for practical engineering applications. This will enhance the understanding on the thermomechanical response for the particle reinforced composite materials including the concrete for Civil Engineering structures and other advanced composite materials such as those commonly found in the aerospace applications.

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